

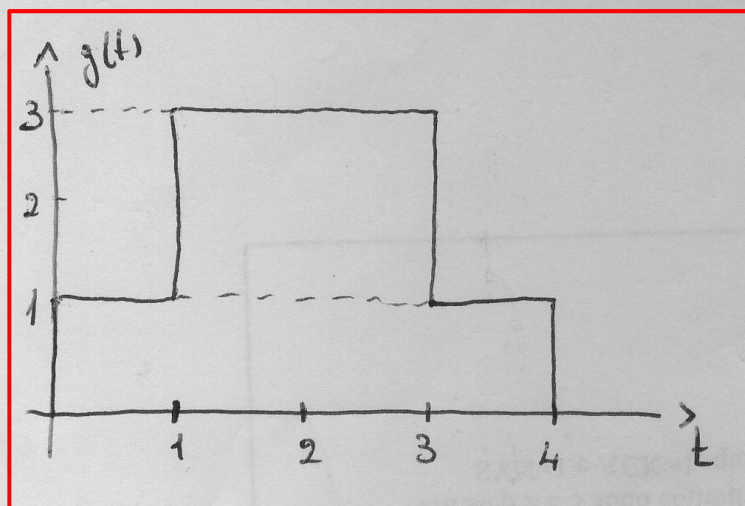
Corso di Teoria dei Segnali

a.a. 2010-2011

Esercitazione n. 3 – Trasformata di Fourier, risposta in
frequenza e campionamento

ESEMPIO :

Volere la TF della funzione $g(t)$ rappresentato :



$$g_1(t) = \pi \left(\frac{t-2}{4} \right)$$

↓ \mathcal{F}

$$G_1(f) = 4 \operatorname{sinc}(\pi f 4) e^{-j\omega 2}$$

$$g_2(t) = 2 \pi \left(\frac{t-2}{2} \right)$$

↓ \mathcal{F}

$$G_2(f) = 2 \cdot 2 \operatorname{sinc}(\pi f 2) e^{-j\omega 2}$$

$$G(f) = G_1(f) + G_2(f) \Rightarrow G(f) = 4 [\operatorname{sinc}(4\pi f) + \operatorname{sinc}(2\pi f)] e^{-j4\pi f}$$

ESEMPI (X C ASA) :

$$\mathcal{F} \left[\begin{cases} e^{-\alpha t} & t \geq 1 \\ 0 & t < 0 \end{cases} \right]$$

$$\mathcal{R}. \frac{e^{-(\alpha + j2\pi f)}}{\alpha + j2\pi f} = G(f)$$

ESEMPI :

Calcolare la trasformata di Fourier di $g(t) = \begin{cases} At, & 0 < t < T_0 \\ 0, & \text{else} \end{cases}$:

$$G(f) = \int_{-\infty}^{+\infty} At e^{-j\omega t} dt = A \int_{-\infty}^{+\infty} t e^{\alpha t} dt \quad \text{con } \alpha = -j\omega = -j2\pi f, \text{ ricordando}$$

$$\text{che: } \int t e^{\alpha t} dt = e^{\alpha t} \left(\frac{t}{\alpha} - \frac{1}{\alpha^2} \right) \text{ si ottiene:}$$

$$G(f) = A \left[e^{-j\omega t} \left(\frac{t}{-j\omega} - \frac{1}{-\omega^2} \right) \right]_0^{T_0} = A e^{-j\omega T_0} \left(\frac{T_0}{-j\omega} + \frac{1}{\omega^2} \right) - \left(\frac{A}{\omega^2} \right) = A e^{-j\omega T_0} \left(\frac{1}{\omega^2} + \frac{jT_0}{\omega} \right) + \frac{A}{\omega^2}$$

$$\text{con } \omega = 2\pi f$$

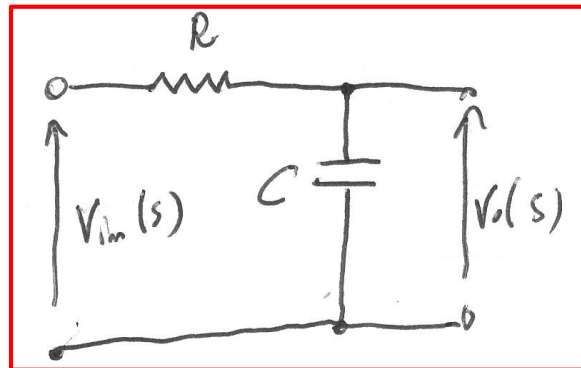
ESEMPI 210: Dato il segnale $g(t)$ con TF: $G(f) = \frac{j2\pi f}{1+j2\pi f}$, trovare
la $X(f)$ per i seguenti segnali:

a) $x(t) = g(2t+2)$: si usa la proprietà di traslazione temporale e di cambiamento di scala: $g(t-T_d) \rightarrow G(f)e^{-j2\pi f T_d}$ e $g(at) \rightarrow \frac{1}{|a|} G(f/a)$, fu cui: $X(f) = \frac{1}{2} \frac{j\pi f}{1+j\pi f} \cdot e^{+j\frac{4\pi f}{2}}$.

b) $x(t) = e^{j2\pi f t} g(t-1)$: si usa la proprietà di traslazione in frequenza:
 $g(t)e^{j2\pi f_c t} \leftrightarrow G(f-f_c)$ con $f_c=1$ nel caso b); $X(f) = \frac{j2\pi(f-1)}{1+j2\pi(f-1)}$.

c) $x(t) = 2 \frac{dg(t)}{dt}$: ci serve la proprietà di derivazione: $d^m g(t)/dt \rightarrow G(f)(j2\pi f)^m$
 $X(f) = 2 \cdot \frac{j2\pi f}{1+j2\pi f} \cdot 2\pi j f = -2 \frac{4\pi^2 f^2}{1+j2\pi f} = -\frac{8\pi^2 f^2}{1+j2\pi f}$

Derivazione delle formule di trasferimento



$$C \Rightarrow q = C V \Rightarrow \frac{dq}{dt} = C \frac{dV}{dt} \Rightarrow \boxed{I = C \dot{V}}$$

$$I \Rightarrow \Phi = L i \Rightarrow \frac{d\Phi}{dt} = L \dot{i} \Rightarrow \boxed{V = L \dot{i}}$$

zero

$$\Rightarrow i(t) = C \frac{dV(t)}{dt} [C]$$

$$v(t) = L \frac{di(t)}{dt} [I]$$

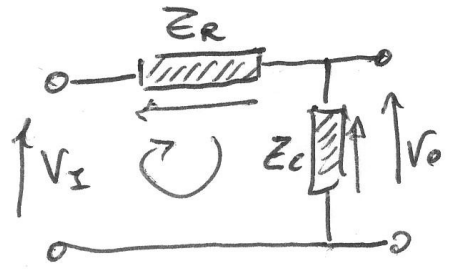
$$\mathcal{F}[i(t)] = C j\omega V(f) = I(f) \quad \text{e} \quad \mathcal{F}[v(t)] = L j\omega I(f)$$

$$I = C \cdot j\omega V \quad (\text{Condensatore}) \quad \text{e} \quad V = L \cdot j\omega I$$

$$V = \underset{\substack{\uparrow \\ \text{impedenza}}}{Z} I \Rightarrow Z = \frac{V}{I}$$

$$c) Z_c = \frac{V}{I} = \frac{1}{j\omega C}$$

$$I) Z_L = \frac{V}{I} = j\omega L$$



$$\Rightarrow V_I - V_R - V_C = 0 \Rightarrow V_I = V_R + V_C$$

$$V_R = Z_R I \quad \text{e} \quad V_C = Z_C I$$

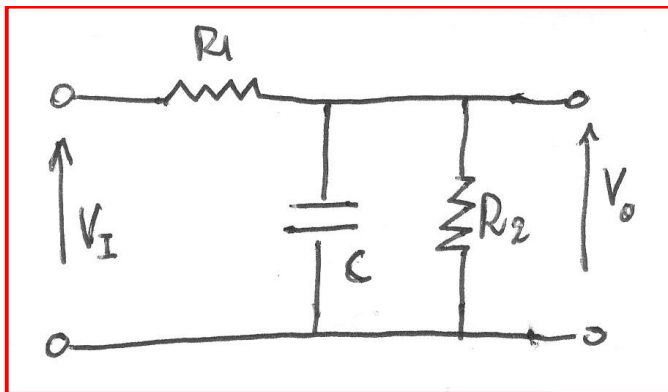
$$V_R = RI \quad V_C = \frac{1}{j\omega C} \cdot I$$

$$V_I = RI + \frac{1}{j\omega C} I = I \cdot \left(R + \frac{1}{j\omega C} \right)$$

$$V_O = V_C = Z_C \cdot I$$

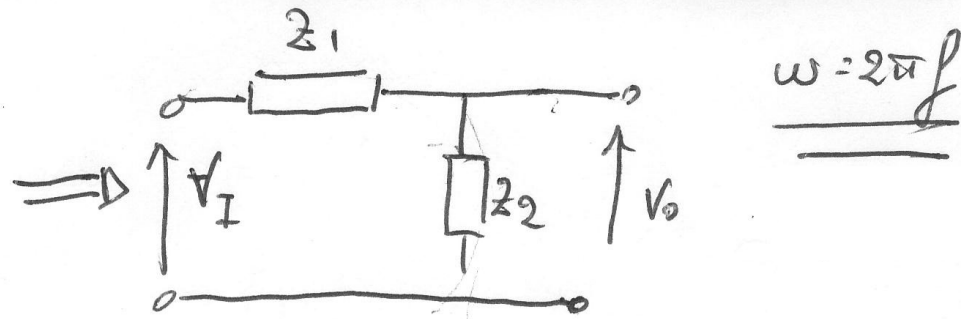
$$\Rightarrow H \Rightarrow \frac{V_O}{V_I} = \frac{\cancel{V_C}}{(R + 1/j\omega C) \cdot \cancel{V_I} \cdot j\omega C} \Rightarrow$$

$$\Rightarrow H = \frac{1}{j\omega RC + 1}$$



$$Z_1 = R_1 \quad Z_2 = C \parallel R_2$$

$$\frac{V_O}{V_I} = \frac{Z_2}{Z_1 + Z_2}$$



$$V_O = Z_2 I = Z_2 \cdot \frac{V_I}{Z_1 + Z_2}$$

SERIE $Z_1 \quad Z_2 \Rightarrow Z_S = Z_1 + Z_2$

PAR. $Z_1 \quad Z_2 \Rightarrow \frac{1}{Z_P} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$$\Rightarrow Z_P = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Z_2 = \frac{Z_C \cdot Z_{R2}}{Z_C + Z_{R2}}$$

$$\frac{V_o}{V_i} = \frac{\frac{Z_C \cdot Z_{R2}}{Z_C + Z_{R2}}}{Z_1 + \frac{Z_C \cdot Z_{R2}}{Z_C + Z_{R2}}}$$

⇓

$$\frac{V_o}{V_i} = H(f) = \frac{\frac{\frac{1}{j\omega C} \cdot R_2}{\left(\frac{1}{j\omega C} + R_2\right)}}{R_1 + \frac{\frac{1}{j\omega C} \cdot R_2}{\frac{1}{j\omega C} + R_2}} = \frac{\frac{R_2}{j\omega C}}{R_1 \left(\frac{1}{j\omega C} + R_2\right) + \left(\frac{1}{j\omega C} \cdot R_2\right)}$$

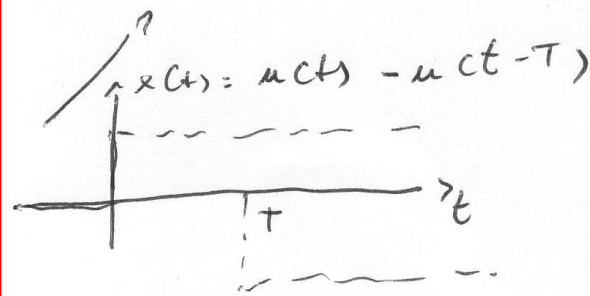
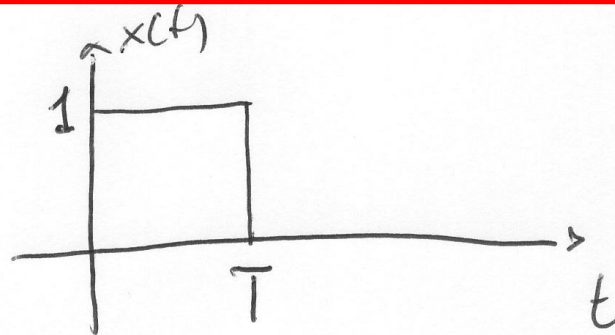
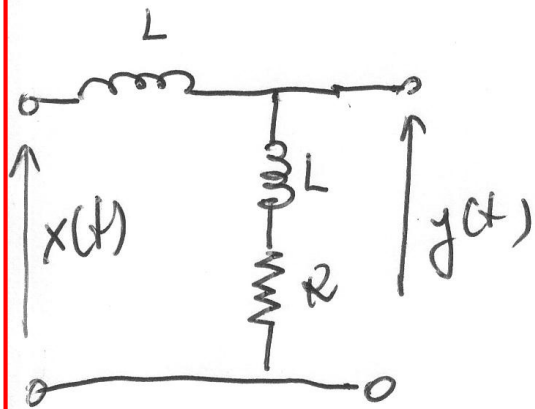
$$= \frac{R_2}{R_1 (1 + R_2 j\omega C) + (R_2)}$$

⇒

$$H = \frac{R_2}{R_1 (1 + R_2 j\omega C) + R_2}$$

ESEMPIO 20)

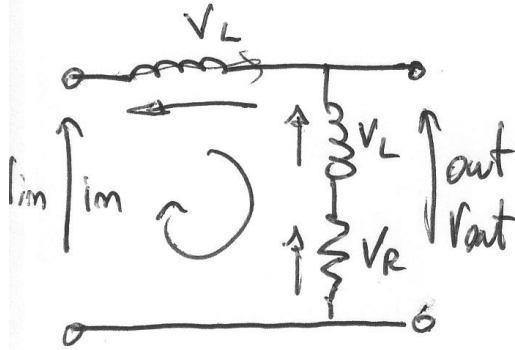
DETERMINARE $y(t)$ SE ALL'INGRESSO È APPLICATO $x(t)$



Per il principio di
sovrapposizione degli
effetti avremo

$$y(t) = g(t) + g(t - T)$$

PROCEDIAMO CALCOLANDO $H(f)$ E POI ANTITRASFORMANDO:



$$V_{IN} - V_L - V_L - V_R = 0$$

$$V_{IN} = 2V_L + V_R = 2j\omega L I_L + R I_R \quad I_L = I_R = I$$

$$\Rightarrow V_{IN} = 2j\omega L I + R I = I (R + 2j\omega L) \quad (1)$$

$$V_{out} = V_L + V_R = j\omega L I + R I = I (j\omega L + R) \Rightarrow$$

$$\Rightarrow I = \frac{V_{out}}{R + j\omega L} \quad \text{CH3 SUBSTITUTION IN (1):}$$

$$V_{IN} = I \cdot (R + 2j\omega L) = V_{out} \cdot \frac{R + 2j\omega L}{R + j\omega L} \Rightarrow$$

$$\Rightarrow \frac{V_{out}}{V_{IN}} = \frac{R + j\omega L}{R + 2j\omega L}$$

$$Z_L = j\omega L$$

$$H(f) = \frac{V_{out}}{V_{IN}} = \frac{R + Z_L}{R + 2Z_L}$$

DIVIDENDO NUMER E DENOMINADOR POR R SIMA

$$H(f) = \frac{1 + j\omega L/R}{1 + 2j\omega L/R} = \frac{1 + j\omega \alpha}{1 + 2j\omega \alpha} = \frac{1/2 + 1/2 + j\omega \alpha}{1 + 2j\omega \alpha} = \frac{1/2}{1 + 2j\omega \alpha} + \frac{1/2}{1 +}$$

$$= \frac{1}{2} \cdot \frac{1}{1 + 2j\omega \alpha} + \frac{2}{2} \cdot \frac{1/2 + j\omega \alpha}{1 + 2j\omega \alpha} = \frac{1}{2} \frac{1}{1 + 2j\omega \alpha} + \frac{1}{2} \cdot \frac{1 + 2j\omega \alpha}{1 + 2j\omega \alpha} = \frac{1}{2} + \frac{1}{2} \frac{1}{1 + 2j\omega \alpha}$$

$$H(f) = \frac{1}{2} + \frac{1}{2} \frac{1}{1 + 2j\omega d}$$

$$Y(f) = H(f)X(f)$$

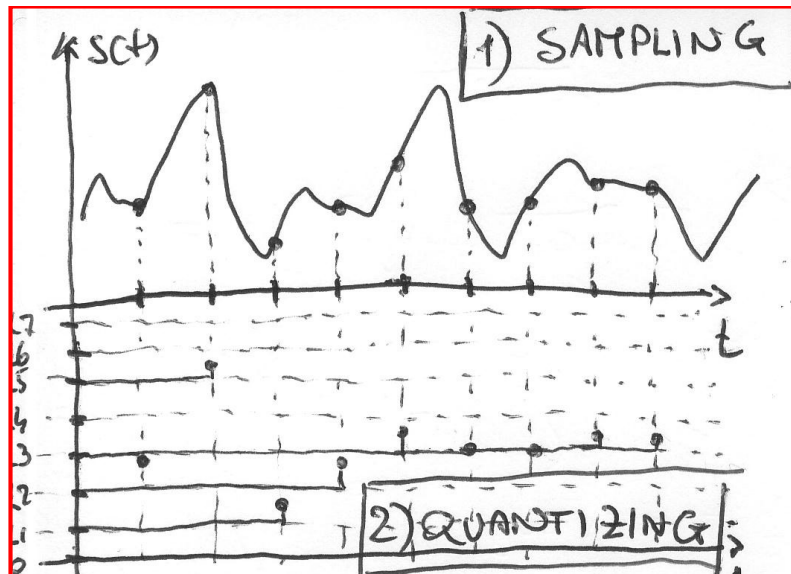
$$X(f) = \frac{1}{2} \delta(f) + \frac{1}{j\omega} - \frac{1}{2} \delta(f) e^{+j2\pi f T/2} - \frac{1}{j2\pi f} e^{j2\pi f T/2} =$$

$$\frac{1}{2} \delta(f) \left[\cancel{1} - e^{j2\pi f T/2} \right] + \frac{1}{j\omega} \left[1 - e^{j2\pi f T/2} \right] \Rightarrow$$

$$\Rightarrow X(f) = [1 - e^{j\omega T/2}] \left[\frac{1}{2} \delta(f) + \frac{1}{j\omega} \right] = [1 - e^{j\omega T/2}] U(f)$$

$$U(f) \triangleq \mathcal{F}[u(t)] = \frac{1}{j2\pi f} + \frac{1}{2} \delta(f) \quad \text{quindi:}$$

$$Y(f) = \left[\frac{1}{2} + \frac{1}{2} \frac{1}{1 + 2j\omega d} \right] [1 - e^{-j\omega T/2}] U(f)$$



3) CODIFYING

$L_0 \equiv 000$ $L_4 \equiv 100$
 $L_1 \equiv 001$ $L_5 \equiv 101$
 $L_2 \equiv 010$ $L_6 \equiv 110$
 $L_3 \equiv 011$ $L_7 \equiv 111$

$L_3 - L_5 - L_1 - L_2 - L_3 - L_3 - L_3 - L_3$



011-101-001-010-011-011-011-011-011

"011101001010011011011011011"

PROMPTORI A

ESEMPIO 21)

DATO UN SEGNALE CHE OCCUPA
UNA BANDA COMPRESA ~~2000~~ IN:

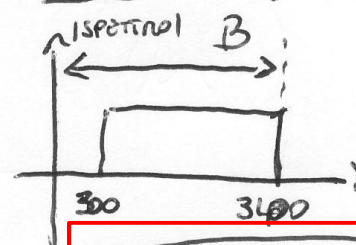
$$[300 \div 3400] \text{ Hz}$$

STABILIRNE LE CARATTERISTICHE
PER POTERNE EFFETTUARE LA
TRASFORMAZIONE DIGITALE:

1) f_s

$$f_{\min} = 2B = 2 \cdot 3400 \text{ Hz} = 6800 \text{ Hz}$$

(samples/s)



FISSIAMO $f_s = 8 \text{ kHz}$

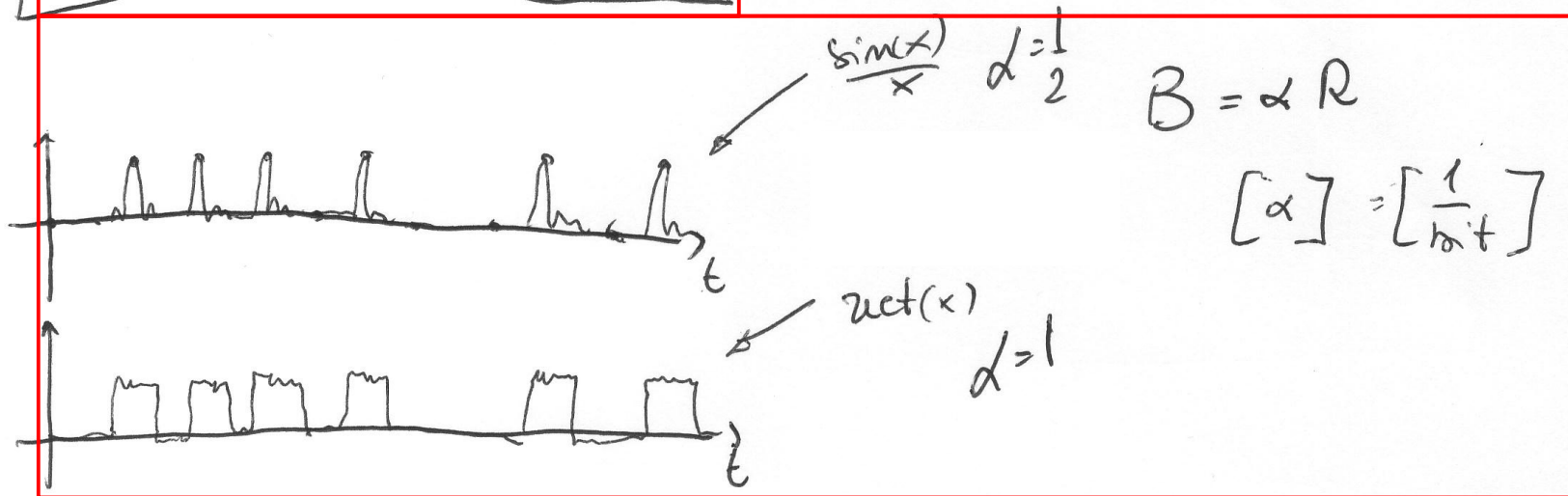
2) IPOTIZZIAMO LA DISPOSIZIONE DI UN QUANTIZZATORE A 3 BIT (8 LIVELLI)

3) CALCOLO DI R:

$$R = m \cdot f_s$$

$$[R] = \left[\frac{\text{bit}}{\cancel{\text{samples}}} \cdot \frac{\cancel{\text{samples}}}{s} \right] = \left[\frac{\text{bit}}{s} \right] = [\text{bps}]$$

$$R = 8000 \cdot 3 = 24 \text{ ~~kB~~ kbps}$$



$$B_{\frac{\sin x}{x}} = \frac{R}{2} = 12 \text{ kHz}$$

$$B_{\text{RECT}(x)} = R = 24 \text{ kHz}$$